Groups & Symmetries

MAT301H1F Fall 2024

1 Welcome

> "*The mathematical sciences particularly exhibit order, symmetry, and limitation; and these are the greatest forms of the beautiful.*"
>
> —Aristotle, *Metaphysica*

From the physical world to the realm of the abstract, symmetry arises everywhere. As the systematic study of symmetry, group theory provides us with a language and the tools to reckon with natural and mathematical patterns. In this course, you will learn the basics of the theory of groups, and discover its applications to mathematics and beyond.

1.1 Teaching Team

Your instructors are:

Shuyang Shen	shuyang.shen@mail.utoronto.ca
Gaurav Patil	g.patil@mail.utoronto.ca

There are also 7 TAs:

Hymn Chan	hymn.chan@mail.utoronto.ca
Jessica Liu	chengjin.liu@mail.utoronto.ca
Coleton Kotch	coleton.kotch@mail.utoronto.ca
Vojin Jovanovic	vojin.jovanovic@mail.utoronto.ca
Mohammad Honari	m.honari@mail.utoronto.ca
John White	jmw3@math.utoronto.ca
Jonathan Pedersen	jsejrp@math.toronto.edu

The TAs will conduct tutorials and some office hours close to the exams. In addition to guiding you through the land of group theory, the TAs will also mark your problem sets and other assessments (see Assessment below).

1.2 Lectures

Both lecture sections will follow the same material at roughly the same pace, and will have the same assessments.

Instructor	Section	Time	Location
Shuyang	LEC0101	Tue 9am–11 am Thurs 9am–10am	KP 108 KP 108
Gaurav	LEC0201	Tue 1pm–3pm Thurs 2pm–3pm	MP 103 MB 128

1.3 Tutorials

TA	Section	Time	Location
Mohammad	TUT0101	Tue 3pm–4pm	OI 5250
Coleton	TUT0201	Wed 12pm–1pm	SS 1088
Coleton	TUT0301	Wed 4pm–5pm	OI 5250
Hymn	TUT0401	Thu 11am–12pm	HA 410
Jonathan	TUT5101	Tue 5pm–6pm	HA 401
Vojin	TUT5201	Wed 5pm–6pm	BA 1230
Vojin	TUT5301	Wed 6pm–7pm	BA 1240

2 Resources

You'll have access to a variety of resources to help you succeed in MAT301.

2.1 Course website

The "MAT301 Groups and Symmetries" course website will be hosted on Quercus. All announcements and course materials will be posted there.

You should check Quercus and your email daily. We recommend that you update your notification settings so that all Quercus announcements are emailed to you.

2.2 Piazza

We will be using Piazza as a discussion forum for this course. If you're stuck on a homework problem, or have a question about a quiz, or need clarification,

please ask us via Piazza. Members of the teaching team will regularly check the forum to answer your questions and address your concerns.

We strongly encourage you to engage with your fellow classmates through Piazza. Adjacent topics or puzzles you may wish to discuss go here. If you spot a question you can answer, please do! Also, feel free to discuss homework problems, but please avoid posting explicit solutions before the due date; we will delete such posts (see Academic Integrity below).

2.3 Email

Email should be reserved for administrative purposes. Due to the size and nature of the course, we will not be answering emails about math; math questions should go on Piazza.

If you need to contact one of us directly, you may send us an email. Please include "MAT301" in the subject line.

2.4 Office hours

Name	**Time**	**Location**
Gaurav	Fri 4pm–5pm	TBA
Shuyang	Tue 12pm–1pm	TBA
Jonathan	TBA	TBA
Mohammad	TBA	TBA

There will be additional office hours before the Midterm and Final Exam.

2.5 Textbook

There is no required textbook for this course. We will follow and post our own lecture notes instead.

However, we recommend a few textbooks as supplementary material:

- *Contemporary Abstract Algebra* by Joseph Gallian
- *Abstract Algebra* by David Dummit and Richard Foote
- *A Course on Group Theory* by John Rose
- *Algebra* by Michael Artin

- *Topics in Algebra* by I.N. Herstein

These textbooks are great sources of exercises, and we may suggest problems from them every now and then.

3 Content

MAT301 is a third-year math course that combines theory and computation. While the emphasis will be on calculations and concrete examples, proofs and abstract constructions *will* show up in lectures and on assessments.

3.1 Learning Goals

Throughout this course, you will:

i) internalize the essence of the abstract group concept,

ii) recognize the prevalence of symmetry in mathematics,

iii) gain experience with fundamental group-theoretic constructions,

iv) translate between various representations of groups and their elements,

v) compute and interrelate several cardinal invariants associated to groups, and

vi) continue to hone your skills as communicators of mathematics.

3.2 Prerequisites

While anyone can learn the basics of group theory with enough time and patience, we expect you to be familiar with the material normally covered in the prerequisite courses. In particular, you should be comfortable with:

- linear algebra (as in MAT 224 or MAT 247)
- calculus (as in MAT 235, MAT 237, or MAT 257)
- proofs (as in MAT 157, MAT 246, CSC 236, or CSC 240)

We've prepared a diagnostic quiz for you to self-assess your readiness. Please take the quiz and brush up on anything you need to.

3.3 Schedule

Here's a rough schedule of the topics we'll be covering in the course. There may be some give or take from week to week.

Week of	nº	Topics
Sept 3	1	Syllabus, Introduction to groups with examples, Set theory, Functions, Cartesian Products, Equivalence classes
Sept 10	2	Formal Definition of a Group, Group actions, Vocabulary and examples of groups
Sept 17	3	Subgroups and Cosets, Orders, Lagrange's Theorem, Congruence Groups (+ and $\times$)
Sept 24	4	Cyclic Groups, Dihedral Groups, Conjugation
Oct 1	5	Normal Subgroup and Quotient groups
Oct 8	6	Symmetric groups and Alternating groups, Cayley's Theorem
Oct 15	7	Morphisms, Kernel and image, Semi-direct product
Oct 22	8	Group actions, Orbits, Stabilizers, Class Equation
Oct 28 - Reading week		
Nov 5	9	More group actions
Nov 12	10	Cauchy theorem, Sylow theorems
Nov 19	11	Direct products, Classification of finitely generated abelian groups.
Nov 26	12	Talk about classification of all groups, Review

4 Assessment

4.1 Marking Scheme

Quizzes 5%
Homework .30%
Midterm ...25%
Final Exam 40%

4.2 Quizzes – 5%

We'll have weekly quizzes, available for a one-hour window of your choice on each Friday, administered via Quercus, for a total of about 12. The quizzes will be worth 0.5% each, and only your highest 10 scores will count.

The quizzes are designed to keep you on top of the lecture material. Each quiz will feature a small number of straightforward questions pertaining to the week's lectures.

4.3 Problem sets – 30%

We'll have 6 problem sets in the course, due every other Saturday. Namely:

PS1 due Saturday September 14
PS2 due Saturday September 28
PS3 due Saturday October 12
PS4 due Saturday October 26
PS5 due Saturday November 16
PS6 due Saturday November 30

All deadlines are 11:59pm on the stated due date. Problem sets will be posted on Quercus at least one week before it's due and must be submitted online via Crowdmark. Each problem set will be worth 5%.

A problem set will typically feature 4–6 questions. A few questions will be routine verification pertaining to the most recent couple lectures. Other questions will include more elaborate examples and introduce additional concepts that we didn't have time for in class.

Math is not a spectator sport—just like you can't learn to swim by only watching the Olympics, you also can't learn group theory by only watching us do it. It's important that you get lots of practice doing math, and the problem sets are designed with this goal in mind. Math is ideally viewed as learning a new language that is very far from your base language. Thus, words and grammatical rules do not translate directly from other languages.

Your TAs will mark your submission for clarity of communication as well as mathematical correctness, leaving feedback where appropriate.

4.4 Midterm and Final Exam – 25% and 40%, respectively

We will have one midterm and one final exam in this course.

The **Midterm** is scheduled on

Friday October 18th 4–6pm.

If you cannot make it to the midterm, you must contact us before October 19th 4pm. If your request is approved, you may receive an accommodation. The accommodation to be used will be decided by your instructor. Some examples of accommodations may include: an oral exam, a written make-up test, or a re-weighting of your assessments.

The Final Exam will be scheduled sometime during the exam period between Dec 6 and Dec 23. The precise date will be announced by the Faculty of Arts and Science.

The Midterm and Final Exam will incorporate ideas and concepts in the classes as well as those on problem sets. So ideally, these will be based off previously seen problems, and thus should be easier than the problem sets. If, however, you are not comfortable with the tricks and techniques introduced in the problem sets, these tests will seem more difficult. The point is there is incentive to focus on the concepts and solutions of the problem sets, even after you receive your grades for them. We recommend you take the time to understand the details of the solutions to each problem in the problem set. One of the most underrated techniques of study is to first concretely verbalize the new trick or concept that you have learnt in a problem, and then design the hardest possible problem you can come up with using said concept. We suggest you share such problems with one another on piazza to enhance your understanding.

5 Policies

5.1 Late work

Missed quizzes will receive a grade of 0 with no opportunity for makeups. As for problem sets: if it's a day late, we'll subtract 20%, and if it's two days

late, we'll subtract 50%. Problem set submissions that are more than two days late will not be accepted.

5.2 Regrade requests

Regrade requests for any piece of term work should be made within **one week** of the date the grade is returned. They must be made to the TA in charge (the TA in charge of a question will be communicated to you) of the question, the subject of the email should always be as follows:

MAT301 Regrade Request-PS2 Q5

If a regrade is granted, the student must accept the resulting grade as the new grade, whether it goes up or **down** or remains the same.

5.3 Extenuating circumstances

Should you be unable to complete coursework in time for a legitimate reason (e.g. illness, bereavement, other emergencies), let us know via email and we'll try to work something out. Students are now allowed to self-declare illness on acorn.

6 Accessibility

Students with diverse learning styles and needs are welcome in this course. In particular, if you have a disability/health consideration that may require accommodations, please feel free to approach us and/or Accessibility Services at (416) 978-8060: studentlife.utoronto.ca/as.

7 Academic integrity

Academic integrity is a fundamental value of learning and scholarship at the UofT. Participating honestly, respectfully, responsibly, and fairly in this academic community ensures that your UofT degree is valued and respected as a true signifier of your individual academic achievement.

The University of Toronto's Code of Behaviour on Academic Matters outlines the behaviours that constitute academic misconduct, the processes for

addressing academic offences, and the penalties that may be imposed. The Code is the official rulebook for academic behaviour at the University of Toronto, and you are expected to know the rules.

The University of Toronto treats cases of academic misconduct very seriously. All suspected cases of academic dishonesty will be investigated following the procedures outlined in the Code. If you have any questions about what is or is not permitted in this course, please do not hesitate to contact us. If you are experiencing personal challenges that are having an impact on your academic work, please speak to one of us or seek the advice of your college registrar.